

on the method of successive approximations. Comparison shows that the disagreement between the results equals on the average $\pm 8\%$. The dependences shown in Fig. 4 are generalized by the formula

$$\xi = 0.03(1.02 - E^{0.5}) + (4.7 - 3.2 \lg E) Re_1 \cdot 10^{-6}. \quad (4)$$

The losses of head in descending dispersed-annular flow can be easily determined with the help of the dependences (1), (3), and (4) obtained above.

NOTATION

d and l , diameter and length of the pipe; ρ , density; μ , dynamic viscosity; w , velocity; g , acceleration of gravity, σ , surface tension; Γ , mass density of irrigation; $J = \Gamma(\pi d l_0)^{-1}$, specific loss intensity; $l_0 = 1$ m; E , relative mass fraction of loss; c , mass concentration of drops in the gas volume; ΔP , friction losses; τ , tangential stress on the interface; ξ , resistance coefficient; δ , thickness of the film; $Re_1 = 4\Gamma\mu_1^{-1}$; $Re_2 = w_2\rho_2\mu_2^{-1}$; $Fr_2 = w_2^2(gd)^{-1}$; $\delta^* = \delta(gp_1^2\mu_1^2)^{1/3}$; $\tau^* = \tau\delta\sigma^{-1}$. Indices: 1, liquid; 2, gas.

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EFFECT OF TEMPERATURE-INDUCED PHASE SEPARATION IN TWO-PHASE FLOWS

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The results of a theoretical study of the fact that on expansion of an adiabatic two-phase flow in a nozzle the stagnation temperature of one of the phases rises above while that of the other drops below the initial value are presented.

Temperature-induced phase separation occurs with adiabatic expansion of a stationary two-phase flow in a nozzle. If after expansion the phases are stopped and rapidly separated, it is possible to obtain two systems with substantially different temperatures. This effect was confirmed experimentally by Stolyarov [1, 2], who performed experiments on the expansion of a mixture of compressed air with finely dispersed liquid particles in a nozzle. In the case of particles of water after stagnation and separation of the phases artificial snow formed at the outlet. For expansion of air with kerosene or a water solution of diethylene glycol, after stagnation and separation of the liquid phase, the temperature of the liquid phase was lower than 0°C . An analogous effect is the basis for the so-called "snow gun" [3]. According to the experimental data, the temperature-induced phase separation permits obtaining quite low temperatures and the effect can be employed for refrigeration. However, the well-known theoretical investigations of two-phase flows [4-7] do not contain an analysis of the physical essence of this effect. The purpose of this work is to give a theoretical description of the temperature-induced phase separation in a two-phase jet.

We shall first study the equilibrium adiabatic flow of a two-phase medium for which the static (thermodynamic) temperatures of the phases and their velocities are identical. We shall make the standard assumptions that the heat capacity of the liquid (solid) and gaseous

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phases are constant, the specific volume of the liquid (solid) phase is constant, and we shall neglect the thermodynamic effects caused by dispersion of the liquid (solid) phase in the flow and evaporation or condensation. In this case the stagnation temperature of the phases are different:

$$T_{a0} = T + \frac{w^2}{2c_{pa}}; \quad (1)$$

$$T_{b0} = T + \frac{w^2}{2c_b} + \frac{\Delta p}{\rho_b c_b}. \quad (2)$$

The term $\Delta p/\rho_b c_b$ in the formula (2) is usually neglected. Therefore $T_{a0} > T_{b0}$, if $c_{pa} < c_b$; $T_{a0} < T_{b0}$ if $c_{pa} > c_b$; and $T_{a0} = T_{b0}$ if $c_{pa} = c_b$. So, the stagnation temperatures of the phases in an equilibrium two-phase flow are different, if the heat capacity of the phase are different.

It is obvious that in the process of expansion the total energy of the gas changed by the amount

$$\Delta E_a = G_a \left[i_{af} - i_{ai} + \frac{w^2}{2} \right] = G_a \left[c_{pa} T_f - c_{pa} T_i + \frac{w^2}{2} \right],$$

while the total energy of the particles changed by the amount

$$\Delta E_b = G_b \left[i_{bf} - i_{bi} + \frac{w^2}{2} \right] = G_b \left[c_b T_f - c_b T_i + \frac{w^2}{2} \right].$$

In these equations and below we shall neglect the small terms $\Delta p/\rho_b$ and we shall assume that $c_{pa} = \text{const}$ and $c_b = \text{const}$. Since the expansion occurs without work being performed, the total energy of the gas and the particles remains unchanged, and therefore

$$\Delta E_a + \Delta E_b = 0$$

or

$$G_a c_{pa} (T_f - T_i) + G_a \frac{w^2}{2} + G_b c_b (T_f - T_i) + G_b \frac{w^2}{2} = 0. \quad (3)$$

From the formula (3) we find the kinetic energy per unit mass:

$$\frac{w^2}{2} = \frac{(G_a c_{pa} + G_b c_b)(T_i - T_f)}{G_a + G_b} = [\xi c_{pa} + (1 - \xi) c_b] (T_i - T_f).$$

Then we can determine the difference of the final and initial stagnation temperatures of the gas:

$$T_{a0} - T_i = T_f - T_i + \frac{w^2}{2c_{pa}} = (1 - \xi) \left(\frac{c_b}{c_{pa}} - 1 \right) (T_i - T_f). \quad (4)$$

In an analogous manner we can determine also the difference of the temperatures for the particles:

$$T_{b0} - T_i = T_f - T_i + \frac{w^2}{2c_b} = \xi \left(\frac{c_{pa}}{c_b} - 1 \right) (T_i - T_f). \quad (5)$$

Analyzing Eqs. (4) and (5) we can draw the following conclusions: the final stagnation temperatures of the phases are in general different and depend on the ratio of the heat capacities of the phases.

At the same time, if $c_b/c_{pa} > 1$, then $T_{a0} > T_i$ and $T_{b0} < T_i$, which is observed in experiments on expansion of two-phase jets of air with finely dispersed particles of water, kerosene, or diethylene glycol [1, 2]. In the case when $c_b/c_{pa} < 1$, $T_{a0} < T_i$ and $T_{b0} > T_i$. Thus, for an equilibrium flow the sign of the temperature-induced phase separation effect is determined by the inequality of their specific heat capacities. This case is analyzed in [8, 9]. It should be noted that the temperature-induced phase separation effect can be used in practice only with rapid stopping and fast separation of the phases. If the stopping and separation of the phases occurs quite slowly, so that thermorelaxation of the phases occurs, the equilibrium temperature of the phases will be close to the initial temperature.

Let us examine the conditions for the appearance of the temperature-induced phase separation effect with a two-temperature and two-velocity flow. We shall make the usual assumptions: the flow is stationary and there is no heat exchange with the walls, the particles are spherical and of the same size, collisions between particles are neglected, the temperature is the same inside the particles, there is no mass transfer in the system, the gas is ideal, the viscosity is taken into account only by interaction forces between phases, there are no external mass forces, and the heat capacities of the phases are constant. We shall study a quasi-one-dimensional two-phase flow in nozzles, where both phases have at the inlet the same thermodynamic temperature T_i and expand from the initial pressure p_i . We shall find the pressure distribution $p(x)$, the distribution of the thermodynamic temperatures of the phases $T_\alpha(x)$ and $T_b(x)$, and the velocity distributions of the phases $w_\alpha(x)$ and $w_b(x)$. These functions are determined by the system of equations

$$w_b \frac{dw_b}{dx} = A(w_a - w_b); \quad (6)$$

$$w_b \frac{dT_b}{dx} = D \left[T_a + r \frac{(w_a - w_b)^2}{2c_{pa}} - T_b \right]; \quad (7)$$

$$T_a + \frac{w_a^2}{2c_{pa}} + \frac{1-\xi}{\xi} \frac{c_b}{c_{pa}} T_b + \frac{1-\xi}{\xi} \frac{w_b^2}{2c_{pa}} = C_1; \quad (8)$$

$$\xi w_a \frac{dw_a}{dx} + (1-\xi) w_b \frac{dw_b}{dx} = - \left(\frac{\xi}{\rho_a} + \frac{1-\xi}{\rho_b} \right) \frac{dp}{dx}; \quad (9)$$

$$p = \rho_a R_a T_a, \quad (10)$$

where $\xi = G_a / (G_a + G_b)$ is the relative mass fraction of the gas. In the equation of dynamics (6)

$$A = \frac{3}{8} c_D \frac{\rho_a}{\rho_b} \frac{|w_a - w_b|}{r_b}. \quad (11)$$

Equation (7) describe the heat exchange with a liquid (solid) particle. Unlike the well-known works [4-7] it takes into account the characteristics of heat transfer under conditions of high relative velocities ($w_a - w_b$) of the gas and of the particles. For this reason a dimensionless coefficient describing the restoration of the temperature r is introduced in it, and the computed temperature difference is determined taking into account the kinetic energy of the flow in relative motion: $T_a + r \frac{(w_a - w_b)^2}{2c_{pa}} - T_b$. In addition,

$$D = \frac{3 \text{Nu} \lambda_a}{2c_b \rho_b r_b^2}. \quad (12)$$

Equations (6)-(10) form a nonlinear system, which cannot be solved analytically in the general case. For this reason we shall find analytic solutions for the system of equations for a model case, when the Re numbers are small ($C_D = 24/\text{Re}$, $\text{Nu} = 2$), so that $A = \text{const}$ and $D = \text{const}$, while the velocities w_α and w_b are linear functions of the coordinates: $w_\alpha = \alpha x$, $w_b = \beta x$. From Eq. (10) we find the equation relating α and β [4]:

$$\beta = \frac{1}{2} [\sqrt{A^2 + 4A\alpha} - A]. \quad (13)$$

We substitute the value of the thermodynamic temperature of the gas T_α from the equation of heat transfer (7) into the equation of conservation of energy of the flow (8) and we integrate the linear differential equation so obtained. Taking into account the initial conditions we obtain the distributions of the thermodynamic temperatures of the particles $T_b(x)$ and of the gas $T_\alpha(x)$:

$$T_b(x) = T_i - \frac{\alpha^2 M_B}{2c_{pa}} x^2; \quad (14)$$

$$T_a(x) = T_i - \frac{\alpha^2 M_A}{2c_{pa}} x^2, \quad (15)$$

where

$$M_B = \frac{1 + m_i k^2 - r(1-k)^2}{1 + m_i \frac{c_b}{c_{pa}} + 2 \frac{\beta}{D}};$$

$$M_A = M_B \left(1 + 2 \frac{\beta}{D} \right) + r(1-k)^2;$$

$$m_i = (1 - \xi)/\xi; \quad k = \beta/\alpha.$$

From Eqs. (14) and (15) we find the difference of the stagnation temperatures of the liquid (solid) and gas phases:

$$T_{b0} - T_i = - \frac{1}{2} \left(\frac{\alpha^2 M_B}{c_{pa}} - \frac{\beta^2}{c_b} \right) x^2; \quad (16)$$

$$T_{a0} - T_i = - \frac{\alpha^2}{2c_{pa}} (M_A - 1) x^2. \quad (17)$$

We substitute into the equation of flow dynamics (9) the velocities of the phases $w_a = \alpha x$ and $w_b = \beta x$ and we obtain the following expression:

$$\frac{dp}{dx} = - \frac{[\xi \alpha^2 + (1 - \xi) \beta^2]}{\frac{\xi}{\rho_a} \left(1 + \frac{1 - \xi}{\xi} \frac{\rho_a}{\rho_b} \right)} x. \quad (18)$$

The differential equation (18) is strongly nonlinear. We shall employ the perturbation method to analyze the most common case, when the volume of the liquid (solid) phase is much smaller than the volume of the gas phase $(1 - \xi)/\rho_b \ll \xi/\rho_a$. The approximate solution of Eq. (18) sought can be represented as a sum

$$p(x) = p_0(x) + p_1(x), \quad (19)$$

where $p_0(x)$ is the solution of Eq. (18) with $\alpha = (1 - \xi)/\rho_b = 0$, while $p_1(x)$ is a small perturbation, determined by the fact that $\alpha \neq 0$, $p_1(x) \ll p_0(x)$. Then Eq. (18) implies that

$$\frac{dp_0}{dx} = - \frac{[\xi \alpha^2 + (1 - \xi) \beta^2] \rho_a}{\xi} x; \quad (20)$$

$$\frac{dp_1}{dx} = - \left\{ \frac{[\xi \alpha^2 + (1 - \xi) \beta^2] \rho_a}{\xi} \left(1 - \frac{1 - \xi}{\xi} \frac{\rho_a}{\rho_b} \right) x + \frac{dp_0}{dx} \right\}. \quad (21)$$

Equation (21) is linearized taking into account the fact that $(\alpha \rho_a / \xi) \ll 1$. First we integrate the differential equation (20), substituting into it the value of $T_a(x)$ from (15). Taking into account the initial conditions

$$\text{at } x = 0 \quad p_0(0) = p_i \text{ and } T_a(0) = T$$

we obtain

$$p_0(x) = \frac{p_i}{(T_i/T_a)^m}, \quad (22)$$

where

$$m = \frac{c_{pa}}{R_a} \frac{[\xi \alpha^2 + (1 - \xi) \beta^2]}{\xi M_A \alpha^2}. \quad (23)$$

Substituting (22) into (21) and integrating the equation with the initial condition $p_1(0) = 0$, we find $p_1(x)$. The approximate solution of Eq. (18), obtained by the method of perturbations, has the form

$$p(x) = p_0(x) \left[1 + \frac{p_1(x)}{p_0(x)} \right], \quad (24)$$

where

$$\frac{p_1(x)}{p_0(x)} = \frac{m}{2m-1} [\delta(0) - \delta(x)] - \frac{m}{3m-2} [\delta^2(0) - \delta^2(x)]; \quad (25)$$

$$\delta(x) = \frac{ap_0}{\xi R_a T_a}. \quad (26)$$

The formulas (24) and (25) are valid for $\delta(x) \ll 1$.

We shall use the relations (16) and (17) to analyze the conditions for the appearance of the temperature-induced phase separation effect. In so doing, as follows from the energy equation (8), the temperature differences $T_{b_0} - T_i$ and $T_{a_0} - T_i$ have opposite signs. The formula (16) implies that the temperature difference $T_{b_0} - T_i$ is negative, if (we assume that $r \approx 1$)

$$\frac{c_{pa}}{c_b} - \frac{\alpha^2}{\beta^2} M_B < 0. \quad (27)$$

We substitute into the inequality (27) the expression for M_B and after elementary transformations we find the equivalent inequality

$$\frac{c_{pa}}{c_b} - \frac{m_i - 1 + \frac{2}{k}}{1 + m_i \frac{c_b}{c_{pa}} + 2 \frac{\beta}{D}} < 0,$$

or

$$\frac{c_{pa}}{c_b} - \frac{2 - k}{k + 2 \left(\frac{A}{D} \right) \frac{\beta k}{A}} < 0. \quad (28)$$

In its turn, in accordance with the formula (13), $\beta k/A = 1 - k$. Therefore the inequality (28) assumes the form

$$\frac{c_{pa}}{c_b} \left[k + 2 \left(\frac{A}{D} \right) (1 - k) \right] - 2 + k < 0.$$

For small Re numbers, when $C_D = 24/Re$, $Nu = 2$,

$$\frac{A}{D} = \frac{3}{2} \frac{\mu_a c_{pa}}{\lambda_a} \frac{c_b}{c_{pa}} = \frac{3}{2} Pr_a \frac{c_b}{c_{pa}}.$$

Then the sign of the temperature difference $T_{b_0} - T_i$ is negative, if

$$\frac{1}{3} \left(\frac{c_{pa}}{c_b} + 1 - 3Pr_a \right) k + \left(Pr_a - \frac{2}{3} \right) < 0. \quad (29)$$

It is easily established that the sign of $T_{b_0} - T_i$ is positive if the left side of expression (29) is positive and $T_{b_0} - T_i = 0$ if it equals zero. So, the sign of the temperature difference $T_{b_0} - T_i$ is the same as the sign of the function

$$y = \mathcal{A}_T k + \mathcal{B}_T, \quad (30)$$

where

$$\mathcal{A}_T = \frac{1}{3} \left(\frac{c_{pa}}{c_b} + 1 - 3Pr_a \right); \quad \mathcal{B}_T = Pr_a - \frac{2}{3}; \quad Pr_a = \frac{\mu_a c_{pa}}{\lambda_a}.$$

Thus, if $y \neq 0$ temperature-induced phase separation is observed; if $y = 0$, then $T_{b_0} = T_{a_0}$ and the effect does not occur. In the case when $y < 0$ the liquid (solid) phase cools, while the gas phase is heated. For $y > 0$ the liquid (solid) phase is heated, while the gas phase is cooled. Analysis of the formula (30) shows that:

- 1) the liquid (solid) phase is heated, while the gas is cooled in the following cases:

$$1a) c_{pa}/c_b > 1 \text{ for } Pr_a \geq 2/3, k < 1;$$

$$1b) c_{pa}/c_b < 1 \text{ for } Pr_a > 2/3, k < k_0;$$

$$1c) c_{pa}/c_b = 1 \text{ for } Pr_a > 2/3, k < 1;$$

1d) $c_{pa}/c_b > 1$ for any number Pr_a , $k = 1$ (equilibrium flow);

2) the liquid (solid) phase is cooled, while the gas is heated in the following cases:

2a) $c_{pa}/c_b < 1$ for $Pr_a > 2/3, k_0 < k < 1$;

2b) $c_{pa}/c_b < 1$ for any Pr_a , $k = 1$ (equilibrium flow);

3) the stagnation temperatures of the phases are equal and there is no effect if

3a) $c_{pa}/c_b = 1$ for $Pr_a = 2/3$ and any $k < 1$;

3b) $c_{pa}/c_b = 1$ for any Pr_a , $k = 1$ (equilibrium flow);

3c) $c_{pa} \neq c_b, Pr_a \neq 2/3, k = k_0$.

Here the quantity $k_0 = \beta_T/\alpha_T$ corresponds to the point of inversion $y = 0$.

The theoretical results obtained above were confirmed qualitatively experimentally in [1, 2, 10]. In [10] the results of experiments on the expansion of air with Al_2O_3 particles in a nozzle were studied. The stagnation temperature of the solid phase was essentially measured and it turns out that $T_{b0} - T_i > 0$ - the solid phase was heated. It is easy to establish that in the experiments studied $c_{pa}/c_b > 1, Pr_a > 2/3$. Thus the experiments correspond to the case 1a) and the theoretical prediction agrees with the experimental result.

The experimental results contained in [1, 2] can be analyzed in an analogous manner, and it can be shown that the observed cooling of the liquid phase corresponds to the case 2a).

The previously obtained formulas also enables an approximate calculation of the temperature-induced phase separation effect in real two-phase flows for Reynolds numbers $Re > 1$. In this case the quantities A and D may be assumed to be constant and equal to the average values over the flow and the system of equations can be solved by the iteration method. As a result the distributions of the thermodynamic temperatures $T_a(x)$ and $T_b(x)$, the stagnation temperatures $T_{a0}(x)$ and $T_{b0}(x)$, as well as the pressure $p(x)$ were calculated under conditions corresponding to the data of [1]: $l = 0.17$ m, $p_i = 1.1$ MPa, $p(l) = 0.1$ MPa, $T_i = 283^\circ K$; the working mixture consists of air and a water solution of diethylene glycol (Fig. 1). We assume that $d_b = 10 \mu m$. Figure 1 also shows the experimental values of $T_{a0}(l)$ and $T_{b0}(l)$ taken from the work cited.

NOTATION

T , thermodynamic equilibrium temperature of the mixture; T_a , thermodynamic temperature of the gas phase; T_b , thermodynamic temperature of the liquid (solid) phase; T_{a0} , stagnation temperature of the gas phase; T_{b0} , stagnation temperature of the liquid (solid) phase; w , velocity of the mixture, w_a , velocity of the gas; w_b , velocity of the particles; ρ , density; λ , thermal conductivity; c_{pa} and R_a , specific isobaric heat capacity and the gas constant; c_b , specific heat capacity of the particles; Δp , pressure increment; T_i , initial tempera-

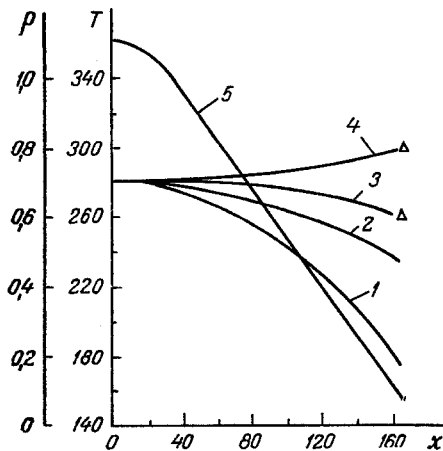


Fig. 1. Distribution of the temperatures $T_a(x)$ (1), $T_b(x)$ (2), $T_{b0}(x)$ (3), $T_{a0}(x)$ (4), and pressure $p(x)$ (5) along the axis of the nozzle (the points are the experimental data). p , MPa; T , $^\circ K$; x , mm.

ture; p_i , initial pressure; x , coordinate; A and D , auxiliary quantities, determined by the relations (10) and (12); r , temperature restoration coefficient; ξ , relative mass fraction of the gas phase; C_1 , a constant; Re , Reynolds number; Pr , Prandtl's number; μ_a , dynamic viscosity of the gas; d_b , r_b , diameter and radius of the particles, respectively; C_D , coefficient of resistance of a spherical particle; Nu , Nusselt number; α and β , coefficients of proportionality of the velocity; m_i , ratio of the mass flow rates of the liquid (solid) and gas phases; k , ratio of the velocities of the liquid (solid) and gaseous phases; $a = (1 - \xi)/\rho_b$; p_0 , pressure for $a = 0$; p_1 , a small pressure perturbation for $a \neq 0$; m and δ , quantities determined by the formulas (23) and (26); y , an auxiliary function, determined by the expression (30); l , length of the nozzle. Indices: a , gas phase; b , liquid (solid) phase; i , initial state; and f , final state.

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FLOW STABILITY OF A FILM OF VISCOUS LIQUID ON THE SURFACE OF A ROTATING DISK

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The stability of a steady axisymmetric flow of a film is studied using the assumption of local plane parallelity. We present the results of numerical calculation.

The flow of a film along the surface of a flat rotating disk is encountered in many technological processes. An example is the preparation of metal powder by the centrifugal method. In the present work we study the linear stability of a steady axisymmetric flow of a film with a relatively small thickness.

Let us suppose that a viscous incompressible liquid is supplied at a constant flow rate near the center of a rotating disk. To describe the flow of the film which is formed on the disk we choose the functions [1]

$$u = \frac{u_r}{\Omega r \delta^2}, \quad v = \frac{1}{\delta^2} \left(\frac{u_\theta}{\Omega r} - 1 \right), \quad w = \frac{u_z}{\Omega H_c \delta^2}, \quad p = \frac{p_f}{\rho \Omega^2 H_c^2},$$

where $\delta = H_c \sqrt{\Omega/\nu}$. The independent variables are chosen as $x = \ln(r/R)$, θ , $y = z/H_c$, $s = \Omega t/\delta^2$.

The functions u , v , w , and p , which depend on x , θ , y , and s and on the form of the free surface of the film $h(x, \theta, s)$, are determined by solving the system of Navier-Stokes equations, the sticking and impermeability conditions on the surface of the disk, the kinematic condition on the free surface where we have also the conditions of zero tangential stress along two directions, and the condition that the stress normal to the surface in the liquid is equal to the stress of capillary forces [1].

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